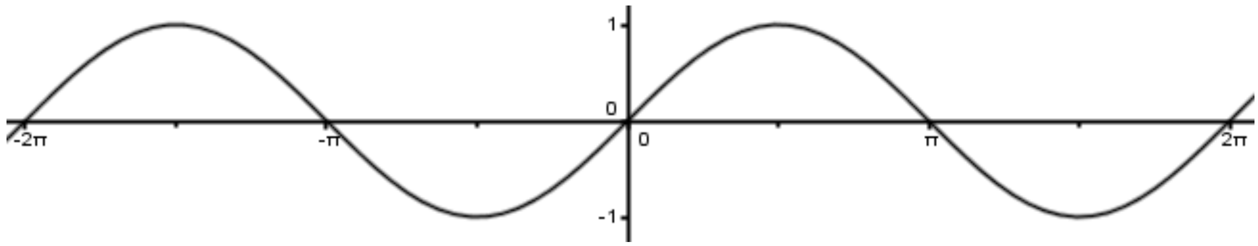
Name:___________________________

SM3 11.1: Inverse Trig Functions

Vocabulary: inverse

Notes: Let's look at the graph of the sine function.

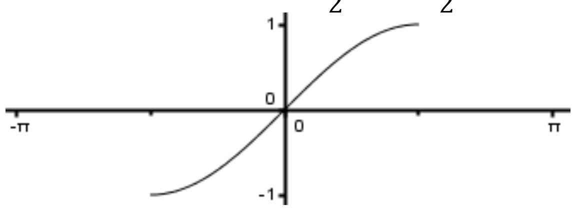


Since the sine function repeats itself, then it is not one-to-one and doesn't have an inverse function. But if we restrict the domain (focus our attention to only a specific interval that passes the horizontal line test), then we can create the inverse of the restricted function. We could choose any interval that is one-to-one, but the easiest is from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

The inverse of the sine function can be written two ways: $\arcsin(x)$ or $\sin^{-1}(x)$ (read as "arc sine" or "sine inverse").

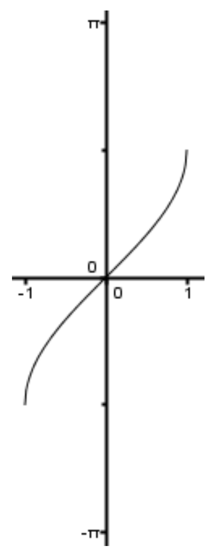
Restricted Sine Function

$$y = \sin x\,, for -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$



Inverse Sine Function

$$y = \sin^{-1} x \;\; or \; y = \arcsin x$$



$\sin^{-1} x$ can be thought of as the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is x.

Example: Find the exact value of $\sin^{-1}\left(\frac{1}{2}\right)$. We start by finding the point on the right half the unit circle (because this corresponds to the angle measures from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$) whose y-coordinate is $\frac{1}{2}$. This corresponds to the angle of $\frac{\pi}{6}$. So the $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

The remaining inverse trig functions are treated the same way.

Restricted Function	Inverse Function	Range for the Inverse
$y = \sin x\,, for -\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$y = \sin^{-1} x$	Right half of the Unit Circle
$y = \cos x\,, for\ 0 \le x \le \pi$	$y = \cos^{-1} x$	Top half of the Unit Circle
$y = \tan x\,, for -\frac{\pi}{2} < x < \frac{\pi}{2}$	$y = \tan^{-1} x$	Right half of the Unit Circle (not including the end points)
$y = \csc x\,, for\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$y = \csc^{-1} x$	Right half of the Unit Circle
$y = \sec x\,, for\ 0 \le x \le \pi$	$y = \sec^{-1} x$	Top half of the Unit Circle
$y = \cot x\,, for\ 0 < x < \pi$	$y = \cot^{-1} x$	Top half of the Unit Circle (not including endpoints)

Evaluate the inverse trig expressions without a calculator. Give answers in radians.

1) $\arcsin(0)$ 2) $\arccos(0)$ 3) $\arctan(0)$ 4) $\text{arccot}(0)$

5) $\arcsin(1)$ 6) $\arccos(1)$ 7) $\arctan(1)$ 8) $\text{arccot}(1)$

9) $\arcsin(-1)$ 10) $\arccos(-1)$ 11) $\arctan(-1)$ 12) $\text{arccot}(-1)$

13) $\arcsin\left(\frac{\sqrt{2}}{2}\right)$ 14) $\arccos\left(\frac{\sqrt{2}}{2}\right)$ 15) $\arctan\left(\frac{\sqrt{3}}{3}\right)$ 16) $\text{arccot}\left(\frac{\sqrt{3}}{3}\right)$

17) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$ 18) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$ 19) $\arctan\left(-\frac{\sqrt{3}}{3}\right)$ 20) $\text{arccsc}\left(-\frac{2\sqrt{3}}{3}\right)$

21) $\arcsin\left(-\frac{1}{2}\right)$ 22) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ 23) $\arctan(-\sqrt{3})$ 24) $\text{arcsec}(-2)$

Evaluate the expressions without a calculator.

25) $\arcsin\left(\sin\left(\frac{\pi}{3}\right)\right)$ 26) $\arccos\left(\cos\left(\frac{5\pi}{4}\right)\right)$ 27) $\arctan\left(\tan\left(\frac{\pi}{6}\right)\right)$

28) $\sin\left(\arcsin\left(-\frac{1}{2}\right)\right)$ 29) $\cos\left(\arccos\left(-\frac{1}{2}\right)\right)$ 30) $\tan\left(\arctan\left(-\frac{1}{2}\right)\right)$

31) $\arcsin\left(\sin\left(\frac{4\pi}{3}\right)\right)$ 32) $\arccos\left(\cos\left(\frac{3\pi}{2}\right)\right)$ 33) $\arctan\left(\tan\left(\frac{5\pi}{6}\right)\right)$

34) $\sin\left(\arccos\left(\frac{1}{2}\right)\right)$

35) $\cos\left(\arcsin\left(\frac{\sqrt{2}}{2}\right)\right)$

36) $\tan\left(\operatorname{arccot}\left(\sqrt{3}\right)\right)$

37) $\arcsin\left(\cos\left(2\arccos\left(\sin\left(\frac{\pi}{6}\right)\right)\right)\right)$

38) $\arcsin\left(\frac{1}{2}\cot\left(\arccos\left(\frac{1}{2}\tan\left(\frac{\pi}{3}\right)\right)\right)\right)$

39) $\operatorname{arccot}\left(\sin\left(2\arctan(\cos(\pi))\right)\right)$

40) Why would it be a poor decision for the function $y = \arccos(x)$ to have a range of $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$?